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Single-Inclusive hadron production in polarized pp scattering at next-to-leading logarithmic accuracy

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Abstract

We study the resummation of large logarithmic perturbative corrections to the partonic cross sections relevant for the process $pp \to hX$ at high transverse momentum of the hadron h, when the initial protons are longitudinally polarized. We perform the resummation to next-to-leading logarithmic accuracy. We present numerical results for center-of-mass energies $\sqrt{S} = 19.4$ GeV, relevant for comparisons to data from the Fermilab E704 experiment, and $\sqrt{S} = 62.4$ GeV, where preliminary data from RHIC have recently become available. We find significant enhancements of the spin-dependent cross sections, but a decrease of the double-spin asymmetry for the process. This effect is less pronounced at the higher energy.

1 Introduction

The spin structure of the nucleon continues to be a particular focus of modern nuclear and particle physics. As is well known, the total quark and anti-quark (summed over all flavors) spin contribution to the nucleon spin was found to be only about $\sim 25\%$, implying that the gluon spin contribution and/or orbital angular momenta may play an important role. There is currently much experimental activity aiming at further unraveling the nucleon's spin structure. One emphasis is on the determination of the spin-dependent gluon distribution, Δg, of the nucleon, which ultimately would give the gluon spin contribution to the nucleon spin. Deep-Inelastic scattering (DIS) has provided most of the presently available information on nucleon spin structure, but has left Δg essentially unconstrained [1, 2, 3]. Particularly good prospects for determining $\Delta g(x, Q^2)$ over a wide range of momentum fractions x and scales Q are offered at the Relativistic Heavy-Ion Collider (RHIC) at BNL, which is the first polarized proton-proton collider. Spin asymmetries in high-energy pp scattering can be particularly sensitive to Δg, for processes where gluons in the initial state contribute already at the lowest order of perturbation theory [4]. One example is the single-inclusive production of large transverse-momentum (p_T) hadrons, $pp \to hX$. Indeed, RHIC data taken at $\sqrt{S} = 200$ GeV on the double-spin asymmetry A_{LL} for $pp \to \pi X$ [5] and for the related process $pp \to \mathrm{jet}X$ [6] are now starting to put significant constraints on Δg, indicating that Δg is not too sizable in the accessed region of gluon momentum fractions. Similar conclusions are drawn from results obtained in lepton scattering [7].

RHIC is, however, not the first place where the spin asymmetry A_{LL}^{π} for $pp \to \pi X$ was investigated. The Fermilab E704 fixed-target experiment presented measurements of A_{LL}^{π} for 200 GeV protons impeding on a proton target [8], resulting in $\sqrt{S} = 19.4$ GeV center-of-mass (c.m.) energy. An asymmetry consistent with zero was found for pions produced with transverse momenta $1 \leq p_T \leq 4$ GeV at central c.m. system angles. An interesting question is whether this information already puts a constraint on Δg at the x values relevant here, $0.1 \lesssim x \lesssim 0.4$. In [8] the experimental data were also compared to theoretical leading-order (LO) calculations using various different Δg distributions. It was found that indeed there was some sensitivity of the data to Δg, with extremely large Δg (of size similar to the unpolarized gluon distribution in the accessed x region) seemingly ruled out. On the other hand, there are arguments against such a direct interpretation. For typical fixed-target kinematics as those in the E704 experiment, unpolarized single-inclusive hadron cross section data are generally not at all described even by next-to-leading order (NLO) (let alone, LO) theoretical calculations [9], with theory falling way short. One may therefore wonder if it is then sensible to confront LO calculations for A_{LL}^{π} with the data. In order to address this issue, Ref. [10] considered the effects of possible Gaussian-distributed "intrinsic" transverse momenta (k_T) of the partons on A_{LL}^{π} in this kinematic regime. It was found that intrinsic k_T tends to decrease the spin asymmetry significantly, so that even relatively large Δg appeared to be compatible with the E704 data. At the same time, intrinsic k_T improves the comparison with the unpolarized cross section data. However, from a theoretical point of view, implementation of intrinsic k_T into a single-inclusive cross section is not really a satisfactory approach because one only has *collinear* factorization in this case. At best, intrinsic k_T effects may be regarded as providing an effective model for possible power-suppressed contributions to the cross section. Implementation of intrinsic k_T also obscures the role of perturbative higher-order contributions to the cross section. Nonetheless, the results of [10] indicate that there can be substantial contributions to A_{LL}^{π} in the fixed-target regime that go beyond low orders of perturbation theory.

Progress on the theoretical description of the unpolarized single-inclusive hadron cross section in the fixed-target energy regime was made in Ref. [11]. For typical fixed-target kinematics, the value of $x_T \equiv 2p_T/\sqrt{S}$ is relatively large, $x_T \gtrsim 0.1$. It turns out that the partonic hard-scattering cross sections relevant for $pp \to hX$ are then largely probed in the "threshold"-regime, where the initial partons have just enough energy to produce the high-transverse momentum parton that subsequently fragments into the hadron, and its recoiling counterpart. Relatively little phase space is then available for additional radiation of partons. In particular, gluon radiation is inhibited and mostly constrained to the emission of soft and/or collinear gluons. The cancellation of infrared singularities between real and virtual diagrams then leaves behind large double- and single-logarithmic corrections to the partonic cross sections. These logarithms appear for the first time in the NLO expressions for the partonic cross sections, where they arise as terms of the form $\alpha_S \ln^2(1-\hat{x}_T^2)$ in the rapidity-integrated cross section, where $\hat{x}_T \equiv \hat{p}_T/\sqrt{\hat{s}}$ with $\hat{p}_T$ the transverse momentum of the produced parton and $\hat{s}$ the c.m. energy of the initial partons. At yet higher (kth) order of perturbation theory, the double-logarithms are of the form $\alpha_S^k \ln^{2k}(1-\hat{x}_T^2)$. When the threshold regime dominates, it is essential to take into account the large logarithms to all orders in the strong coupling α_S, a technique known as "threshold resummation" [12]. Based on earlier work [13, 14] on the resummation for $2 \to 2$ QCD hard-scattering, we examined the effects of threshold resummation on the single-inclusive hadron cross section in [11] and found very significant enhancements of the theoretical prediction in the fixed-target regime, which in fact lead to a relatively good agreement between resummed theory and the data. This also sheds light on the size of additional power-suppressed contributions to the cross sections (among them, perhaps, effects related to intrinsic k_T; see also Ref. [15]), which do not seem to play a dominant role. We concluded that threshold resummation is an essential part of the theoretical description in the typical fixed-target kinematic regime. Its effects at higher energies (such as at RHIC) are much smaller, even though it has to be said that one is here typically much further away from the threshold regime so that the applicability of threshold resummation is not entirely clear then.

In the light of the results of Ref. [11] and the E704 data, it appears desirable to apply threshold resummation also to the spin asymmetry A_{LL}^{π}, which is the goal of this paper. In this way, one may hope to put the theoretical description of A_{LL}^{π} for single-inclusive hadron production in the fixed-target regime on firmer ground. One may then also revisit the question as to whether the E704 data already allow to put a constraint on Δg.

We also note that recently preliminary data for the cross section and double-spin asymmetry taken at RHIC's lower energy $\sqrt{S} = 62.4$ GeV have been reported [16, 17]. Even though the approximations needed for threshold resummation to be useful work slightly worse for the kinematics relevant here, it is of great interest to confront the resummation with the data. This will also be done in this paper.

The remainder of this paper is organized as follows: Section 2 summarizes the theoretical perturbative-QCD framework for the process under study. In Sec. 3, we present the resummed spin-dependent cross section to next-to-leading logarithmic (NLL) accuracy. Section 4 gives phenomenological results for the effects of threshold resummation on the spin-dependent high-p_T pion cross section at $\sqrt{S} = 19.4$ GeV and at $\sqrt{S} = 62.4$ GeV, and on the corresponding double-spin asymmetries A_{LL}^{π}. Finally we draw our conclusions in Sec. 5. Two Appendices contains the relevant ingredients for the resummation in the spin-dependent case.

2 Cross section and spin asymmetry in perturbation theory

We are considering the process

$$p(p_1, \Lambda_1) + p(p_2, \Lambda_2) \longrightarrow h(p_3) + X\,, \tag{2.1}$$

where the Λ_i denote the helicities of the initial protons, and the p_i ($i = 1, 2, 3$) are the four-momenta of the "observed" hadrons. One defines the spin-averaged and spin-dependent cross sections as

$$\begin{aligned} d\sigma &= \frac{1}{2}\left[d\sigma(\Lambda_1 = +, \Lambda_2 = +) + d\sigma(\Lambda_1 = +, \Lambda_2 = -)\right]\,, \\ d\Delta\sigma &= \frac{1}{2}\left[d\sigma(\Lambda_1 = +, \Lambda_2 = +) - d\sigma(\Lambda_1 = +, \Lambda_2 = -)\right]\,, \end{aligned} \tag{2.2}$$

respectively, and their double-spin asymmetry as

$$A_{LL} = \frac{d\Delta\sigma}{d\sigma}\,. \tag{2.3}$$

Hadron h is assumed to be produced at large transverse momentum p_T. For such a large-momentum-transfer reaction, the factorization theorem [18] states that cross section may be factorized in terms of collinear convolutions of parton distribution functions for the initial protons, a fragmentation function for the final-state hadron, and short-distance parts that describe the hard interactions of the partons and are amenable to QCD perturbation theory. The long-distance parton distributions and fragmentation functions are universal, i.e., they are the same in any inelastic reaction. Long- and short-distance contributions are separated by a factorization scale.

As discussed in Ref. [11], a major simplification of the resummation formalism occurs when the cross section is integrated over all pseudo-rapidities η of the produced pion. This will also be done in this paper. The factorized spin-dependent cross section for $pp \to hX$ can then be written as

$$\begin{aligned} \frac{p_T^3\, d\Delta\sigma(x_T)}{dp_T} = \sum_{a,b,c} \int_0^1 dx_1\, \Delta f_a\left(x_1, \mu^2\right) \int_0^1 dx_2\, \Delta f_b\left(x_2, \mu^2\right) \int_0^1 dz\, z^2\, D_{h/c}\left(z, \mu^2\right) \\ \times \int_0^1 d\hat{x}_T\, \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta}\, \frac{\hat{x}_T^4\, \hat{s}}{2}\, \frac{d\Delta\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}\,. \end{aligned} \tag{2.4}$$

Here the $\Delta f_{a,b}$ are the spin-dependent parton distributions in the proton,

$$\Delta f_a(x, \mu^2) = f_a^+(x, \mu^2) - f_a^-(x, \mu^2) \tag{2.5}$$

with f_a^+ (f_a^-) denoting the distribution of parton type a with positive (negative) helicity in a proton of positive helicity. $D_{h/c}$ is the fragmentation function for parton c fragmenting into the observed high-p_T hadron. The sum in Eq. (2.4) runs over all partonic channels, with the associated spin-dependent partonic cross sections $d\Delta\hat{\sigma}_{ab\to cX}$. The latter are defined analogously to Eq. (2.2), with helicities now corresponding to parton ones. They are perturbative and have an expansion of the form

$$d\Delta\hat{\sigma}_{ab\to cX} = d\Delta\hat{\sigma}^{(0)}_{ab\to cX} + \frac{\alpha_s}{\pi} d\Delta\hat{\sigma}^{(1)}_{ab\to cX} + \ldots \tag{2.6}$$

with α_S the strong coupling. The scale μ in Eq. (2.4) is the factorization scale. We could distinguish in principle between factorization scales for the initial state (parton distributions) and the final state (fragmentation function). For simplicity, we will not do this in this paper. There is also a renormalization scale, at which the strong coupling constant is evaluated. We will collectively denote all scales by μ. The dependence on μ is implicit in the partonic cross sections in Eq. (2.4). Finally, $\hat{\eta}$ is the pion's pseudorapidity at parton level, related to the one at hadron level by $\hat{\eta} = \eta - \frac{1}{2}\ln(x_1/x_2)$. Its limits are given by $\hat{\eta}_+ = -\hat{\eta}_- = \ln\left[(1+\sqrt{1-\hat{x}_T^2})/\hat{x}_T\right]$ where, as before, $x_T \equiv 2p_T/\sqrt{S}$, and its partonic counterpart is $\hat{x}_T \equiv 2p_T^c/\sqrt{\hat{s}} = x_T/z\sqrt{x_1 x_2}$.

We note that the corresponding expression for the factorized spin-averaged cross section is obtained from Eq. (2.4) by dropping all Δ's, meaning that the spin-dependent parton distributions are replaced by their usual unpolarized counterparts, and the partonic scattering cross sections by the spin-averaged ones.

3 Resummed cross section

As mentioned above, we will follow [11] to perform the threshold resummation only for the case of the fully rapidity-integrated cross section. The resummation of the soft-gluon contributions is achieved by taking a Mellin transform of the cross section in the scaling variable x_T^2:

$$\Delta\sigma(N) \equiv \int_0^1 dx_T^2 \, \left(x_T^2\right)^{N-1} \, \frac{p_T^3 \, d\Delta\sigma(x_T)}{dp_T} \, . \tag{3.7}$$

For the rapidity-integrated cross section, the convolutions in Eq. (2.4) between parton distributions, fragmentation functions, and subprocess cross sections then become ordinary products [11, 19]:

$$\Delta\sigma(N) = \sum_{a,b,c} \Delta f_a(N+1,\mu^2)\, \Delta f_b(N+1,\mu^2)\, D_{h/c}(2N+3,\mu^2)\, \Delta\hat{\sigma}_{ab\to cX}(N) \, , \tag{3.8}$$

where

$$\Delta\hat{\sigma}_{ab\to cX}(N) \equiv \int_0^1 d\hat{x}_T^2 \, \left(\hat{x}_T^2\right)^{N-1} \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta} \, \frac{\hat{x}_T^4 \, \hat{s}}{2} \, \frac{d\Delta\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}} \, . \tag{3.9}$$

In Mellin-moment space, the threshold logarithms become logarithms in the moment variable N. The leading logarithms are of the form $\alpha_S^k \ln^{2k} N$; subleading ones are down by one or more powers of $\ln N$. Threshold resummation results in exponentiation of the soft-gluon corrections in moment space [12, 13]. The leading logarithms are contained in radiative factors for the initial and final partons. Because of color interferences and correlations in large-angle soft-gluon emission at NLL, for QCD hard-scattering the resummed cross section becomes a sum of exponentials, rather than a single one [11, 13], unlike the much simpler cases of the Drell-Yan or Higgs cross sections [12].

Combining results of [12, 13, 20], we can cast the resummed spin-dependent partonic cross

section for each subprocess into a relatively simple form [11]†:

$$\Delta\hat{\sigma}^{(\rm res)}_{ab\to cd}(N) = \Delta C_{ab\to cd}\,\mathcal{D}^a_N\,\mathcal{D}^b_N\,\mathcal{D}^c_N\,J^d_N\,\left[\sum_I \Delta G^I_{ab\to cd}\,\mathcal{D}^{({\rm int})ab\to cd}_{I\,N}\right]\,\Delta\hat{\sigma}^{(\rm Born)}_{ab\to cd}(N)\;, \tag{3.10}$$

where $\Delta\hat{\sigma}^{(\rm Born)}_{ab\to cd}(N)$ denotes the LO term in the perturbative expansion of Eq. (3.9) for each process. We list the moment-space expressions for all the spin-dependent Born cross sections in Appendix A. Each of the functions J^d_N,$\mathcal{D}^i_N$,$\mathcal{D}^{({\rm int})ab\to cd}_{I\,N}$ in Eq. (3.10) is an exponential. $\mathcal{D}^a_N$ represents the effects of soft-gluon radiation collinear to initial parton a and is given, in the $\overline{\rm MS}$ scheme, by

$$\ln\mathcal{D}^a_N = \int_0^1 \frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{(1-z)^2Q^2}\frac{dq^2}{q^2}A_a(\alpha_{\rm S}(q^2))dz\;, \tag{3.11}$$

and similarly for Δ^b_N. Here, $Q^2 = 2p_T^2$. We will specify the function A_a below. Collinear soft-gluon radiation to parton c yields the same function [20]. The function J^d_N embodies collinear, soft or hard, emission by the non-observed recoiling parton d and reads:

$$\ln J^d_N = \int_0^1 \frac{z^{N-1}-1}{1-z}\Big[\int_{(1-z)^2Q^2}^{(1-z)Q^2}\frac{dq^2}{q^2}A_a(\alpha_{\rm S}(q^2)) + \frac{1}{2}B_a(\alpha_{\rm S}((1-z)Q^2))\Big]dz\;. \tag{3.12}$$

Large-angle soft-gluon emission is accounted for by the factors $\mathcal{D}^{({\rm int})ab\to cd}_{I\,N}$, which depend on the color configuration I of the participating partons. A sum over the latter occurs in Eq. (3.10), with $\Delta G^I_{ab\to cd}$ representing a weight for each color configuration, such that $\sum_I \Delta G^I_{ab\to cd} = 1$. Each of the $\mathcal{D}^{({\rm int})ab\to cd}_{I\,N}$ is given as

$$\ln\mathcal{D}^{({\rm int})ab\to cd}_{I\,N} = \int_0^1 \frac{z^{N-1}-1}{1-z}D_{I\,ab\to cd}(\alpha_{\rm S}((1-z)^2Q^2))dz\;. \tag{3.13}$$

Finally, the coefficients $\Delta C_{ab\to cd}$ contain $N-$independent hard contributions arising from one-loop virtual corrections.

Most of the ingredients to Eqs. (3.11)-(3.13) are well-known from the literature because they coincide with the results obtained for spin-averaged scattering. This is the case for the functions A_a, B_a, and $D_{I\,ab\to cd}$, because these are associated with soft gluon emission, which is spin-independent. The only differences between the spin-dependent and the spin-averaged cases reside in the coefficients $\Delta G^I_{ab\to cd}$, $\Delta C_{ab\to cd}$ and of course in the Born cross sections. These terms are all related to hard radiation, which depends on the polarization state and therefore in general differs for the polarized and unpolarized cases.

We first briefly recall the known functions and then turn to the new parts. Each of the functions $\mathcal{F} \equiv A_a$, B_a, $D_{I\,ab\to cd}$ is a perturbative series in $\alpha_{\rm S}$,

$$\mathcal{F}(\alpha_{\rm S}) = \frac{\alpha_{\rm S}}{\pi}\mathcal{F}^{(1)} + \left(\frac{\alpha_{\rm S}}{\pi}\right)^2\mathcal{F}^{(2)} + \ldots\;, \tag{3.14}$$

with [21]:

$$A^{(1)}_a = C_a\;,\quad A^{(2)}_a = \frac{1}{2}\,C_a\left[C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}N_f\right]\;,\quad B^{(1)}_a = \gamma_a\;, \tag{3.15}$$

†Note that the symbols $\mathcal{D}^i_N, \mathcal{D}^{({\rm int})ab\to cd}_{I\,N}$ in the equation below are usually referred to as $\Delta^i_N, \Delta^{({\rm int})ab\to cd}_{I\,N}$ in the literature [11]. We have changed this notation in order to avoid confusion with the label "Δ" indicating spin-dependent cross sections and parton distributions in this paper.

where N_f is the number of flavors, and

$$
\begin{aligned}
&C_g = C_A = N_c = 3 \,, \quad C_q = C_F = (N_c^2-1)/2N_c = 4/3 \\
&\gamma_q = -3C_F/2 = -2 \,, \quad \gamma_g = -2\pi b_0 \,, \quad b_0 = \frac{1}{12\pi}\left(11C_A - 2N_f\right) \,.
\end{aligned}
\tag{3.16}
$$

The coefficients $D^{(1)}_{I\,ab\to cd}$ are listed in Ref. [11].

In order to determine the effects of the color interferences for large-angle soft-gluon emission in the $2\to 2$ processes $ab\to cd$, we can follow the procedures presented in [13, 14]. The soft-anomalous dimensions and soft factors determined in [13] are again identical in the spin-averaged and the spin-dependent cases. The differences arise solely in the color-connected Born cross sections. We therefore only need to derive the latter for polarized scattering, using the same color basis as that chosen in [13]. The results in [13] have actually been given for arbitrary rapidity; for the case of the rapidity-integrated cross section we consider here it is sufficient to set $\hat{\eta}=0$ (see [11] for more detail). Nonetheless, for future convenience, we present in this work the spin-dependent color-connected Born cross sections also at arbitrary rapidity. In this way, they may be directly used in future investigations of the resummed cross section at fixed rapidity. The results are collected in Appendix B. For the case of the rapidity-integrated cross section, the color-connected Born cross sections at $\hat{\eta}=0$, when normalized to the full Born cross section for each partonic channel, give the color weights $\Delta G^I_{ab\to cd}$ [11] that we need to NLL, which are listed in Appendix A.

The perturbative expansion of the coefficients $\Delta C_{ab\to cd}$ reads:

$$
\Delta C_{ab\to cd} = 1 + \frac{\alpha_{\rm S}}{\pi}\Delta C^{(1)}_{ab\to cd} + \mathcal{O}(\alpha_{\rm S}^2) \,. \tag{3.17}
$$

In order to determine the coefficients $\Delta C^{(1)}_{ab\to cd}$, we take advantage of the full analytic NLO calculation of Ref. [22]. For each partonic channel one expands the resummed cross section in Eq. (3.10) to first order in $\alpha_{\rm S}$. Near threshold, one can straightforwardly take Mellin moments of the full NLO expressions of [22]. By comparison of the two results one first verifies that all logarithmic terms in the full NLO results are correctly reproduced by the resummation formalism. The remaining N-independent terms in the NLO cross section give the coefficients $\Delta C^{(1)}_{ab\to cd}$. These turn out to have rather lengthy expressions, and we only give them in numerical form in Appendix A.

This completes the collection of the ingredients for the resummed partonic cross sections. In the exponents, the large logarithms in N occur only as *single* logarithms, of the form $\alpha_{\rm S}^k \ln^{k+1}(N)$ for the leading terms. Next-to-leading logarithms are of the form $\alpha_{\rm S}^k \ln^k(N)$. Knowledge of the coefficients given above allows to resum the full LL and NLL full towers in the exponent. It is useful to expand the resummed exponents to definite logarithmic order:

$$
\ln \mathcal{D}_N^a(\alpha_{\rm S}(\mu^2), Q^2/\mu^2) \;=\; \ln N\; h_a^{(1)}(\lambda) + h_a^{(2)}(\lambda, Q^2/\mu^2) + \mathcal{O}\left(\alpha_{\rm S}(\alpha_{\rm S}\ln N)^k\right) \,, \tag{3.18}
$$

$$
\ln J_N^a(\alpha_{\rm S}(\mu^2), Q^2/\mu^2) \;=\; \ln N\; f_a^{(1)}(\lambda) + f_a^{(2)}(\lambda, Q^2/\mu^2) + \mathcal{O}\left(\alpha_{\rm S}(\alpha_{\rm S}\ln N)^k\right) \,, \tag{3.19}
$$

where $\lambda = b_0\alpha_{\rm S}(\mu^2)\ln N$. The functions $h^{(i)}$ and $f^{(i)}$ have for example been given in [11]. $h^{(1)}$ and $f^{(1)}$ contain all LL terms in the perturbative series, while $h^{(2)}$ and $f^{(2)}$ are only of NLL accuracy. For a complete NLL resummation one also needs the coefficients $\ln \mathcal{D}^{({\rm int})ab\to cd}_{I\,N}$ whose NLL expansion reads:

$$
\ln \mathcal{D}^{({\rm int})ab\to cd}_{I\,N}(\alpha_{\rm S}(\mu_R^2), Q^2/\mu_R^2) \;=\; \frac{D^{(1)}_{I\,ab\to cd}}{2\pi b_0}\;\ln(1-2\lambda) + \mathcal{O}\left(\alpha_{\rm S}(\alpha_{\rm S}\ln N)^k\right) \,. \tag{3.20}
$$

In order to obtain a resummed cross section in x_T^2 space, one needs an inverse Mellin transform. Here one has to deal with the singularity in the perturbative strong coupling constant in Eqs. (3.11)-(3.13), which manifests itself also in the singularities of the functions $h^{(1,2)}$ and $f^{(1,2)}$ above at $\lambda = 1/2$ and $\lambda = 1$. We use the *Minimal Prescription* developed in Ref. [23], which relies on use of the NLL expanded forms Eqs. (3.18)-(3.20), and on choosing a Mellin contour in complex-N space that lies to the *left* of the poles at $\lambda = 1/2$ and $\lambda = 1$ in the Mellin integrand:

$$\frac{p_T^3 \, d\Delta\sigma^{(\mathrm{res})}(x_T)}{dp_T} = \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i} \, \left(x_T^2\right)^{-N} \Delta\sigma^{(\mathrm{res})}(N) \, , \tag{3.21}$$

where $b_0 \alpha_{\mathrm{S}}(\mu^2) \ln C_{MP} < 1/2$, but all other poles in the integrand are as usual to the left of the contour. The result defined by the Minimal Prescription has the property that its perturbative expansion is an asymptotic series that has no factorial divergence and therefore no "built-in" power-like ambiguities.

Finally, in order to make full use of the available fixed-order cross section [22, 24], which in our case is NLO ($\mathcal{O}(\alpha_{\mathrm{S}}^3)$), we match the resummed cross section to the NLO one. We expand the resummed cross section to $\mathcal{O}(\alpha_{\mathrm{S}}^3)$, subtract the expanded result from the resummed one, and add the full NLO cross section:

$$\begin{aligned}\frac{p_T^3 \, d\Delta\sigma^{(\mathrm{match})}(x_T)}{dp_T} = & \sum_{a,b,c} \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i} \, \left(x_T^2\right)^{-N+1} \, \Delta f_{a/P_1}(N,\mu^2) \, \Delta f_{b/P_2}(N,\mu^2) \, D_{c/h}(2N+1,\mu^2) \\ & \times \left[\Delta\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N) - \left. \Delta\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N) \right|_{\mathcal{O}(\alpha_{\mathrm{S}}^3)} \right] + \frac{p_T^3 \, d\Delta\sigma^{(\mathrm{NLO})}(x_T)}{dp_T} \, ,\end{aligned} \tag{3.22}$$

where $\Delta\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N)$ is the polarized resummed cross section for the partonic channel $ab \to cd$ as given in Eq. (3.10). In this way, NLO is taken into account in full, and the soft-gluon contributions beyond NLO are resummed to NLL. Any double-counting of perturbative orders is avoided.

4 Phenomenological Results

We are now in the position to present numerical results for the threshold-resummed spin-dependent cross section and spin asymmetry in single-inclusive pion production in hadronic collisions. We will focus here on $pp \to \pi^0 X$ in fixed-target scattering at $\sqrt{S} = 19.4$ GeV and at $\sqrt{S} = 62.4$ GeV at the RHIC collider. In both these cases, experimental data exist [8, 17].

For our calculations, we need to choose sets of parton distribution and pion fragmentation functions. To study the sensitivity of the measured spin asymmetries to the spin-dependent parton distribution functions, in particular the gluon density Δg, we use the "Glück-Reya-Stratmann-Vogelsang (GRSV)" [1] and the "de Florian-Sassot (DS)" [2] densities. These both offer various sets of distributions, distinguished mostly by Δg distributions of different sizes. The set labeled "GRSV" is the regular ("standard") GRSV set. We will also use the GRSV "max G" set, which has a much larger Δg, given by assuming $\Delta g(x) = g(x)$ at the initial scale for the parton evolution. Likewise, the DS "i+" was constrained to have a much lower Δg than the "iii+" one. For the spin-averaged cross section, we employ the MRST2002 [25] set throughout. The pion

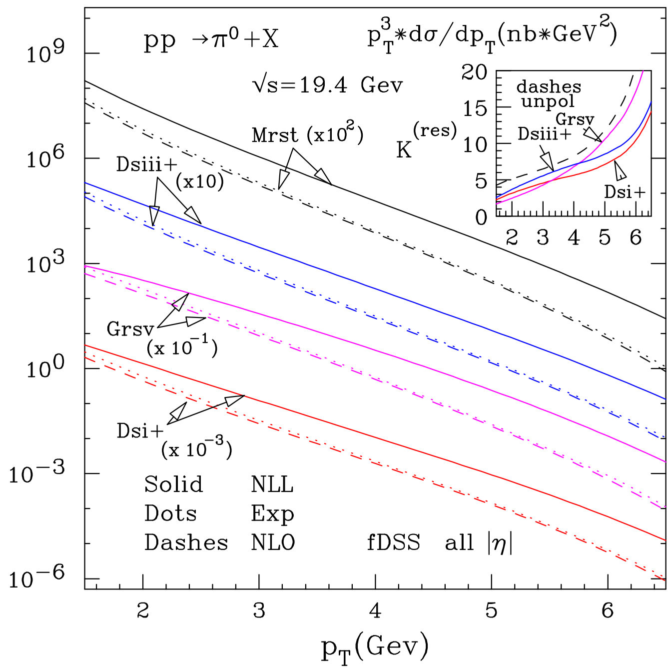

Figure 1: *NLO and NLL resummed cross sections for polarized* $pp \to \pi^0 X$ *at* $\sqrt{S} = 19.4$ *GeV, for various sets of spin-dependent parton distributions of [1, 2]. We also show the* $\mathcal{O}(\alpha_S^3)$ *expansions of the resummed cross sections, and the analogous results in the unpolarized case. For better visibility, we have applied numerical factors to some results, as indicated in the figure. In the upper right inset, we present the ratios between the NLL resummed cross sections and the NLO ones.*

fragmentation functions are taken from the most recent analysis of e^+e^- and pp data, "de Florian-Sassot-Stratmann (fDSS)" Ref. [26]. It is worth noticing that, according to Eq. (3.22), one would like to have the parton densities and fragmentation functions available in Mellin-moment space. Technically, since most of the distributions are only available in x space, we first perform a fit with a simple functional form to the distributions, of which we are able to take moments analytically. This has to be done separately for each parton type and at each factorization scale.

In Fig. 1 we present our results for the spin-dependent cross section at $\sqrt{S} = 19.4$ GeV, integrated over all pion pseudo-rapidities η, for the various sets of the polarized parton distributions of [1, 2]. We have chosen all scales as $\mu = p_T$. We show the full NLO cross section based on the calculations in [22], as well as the NLL resummed predictions. We also display the expansions of the resummed cross section to $\mathcal{O}(\alpha_S^3)$, which is the first order beyond LO. As can be observed, these faithfully reproduce the full NLO result, implying that the threshold logarithms addressed by resummation indeed dominate the cross section in this kinematic regime, so that resummation is expected to be useful. Towards lower $p_T \sim 3$ GeV, the expansions slightly overestimate the NLO cross section, which is expected since one is further away from the threshold regime here, so that the soft-gluon approximation tends to become less reliable. For the sake of completeness the unpolarized case is also represented in Fig. 1.

The inset in Fig. 1 shows the resummed "K-factors" for the cross sections, defined as the ratios

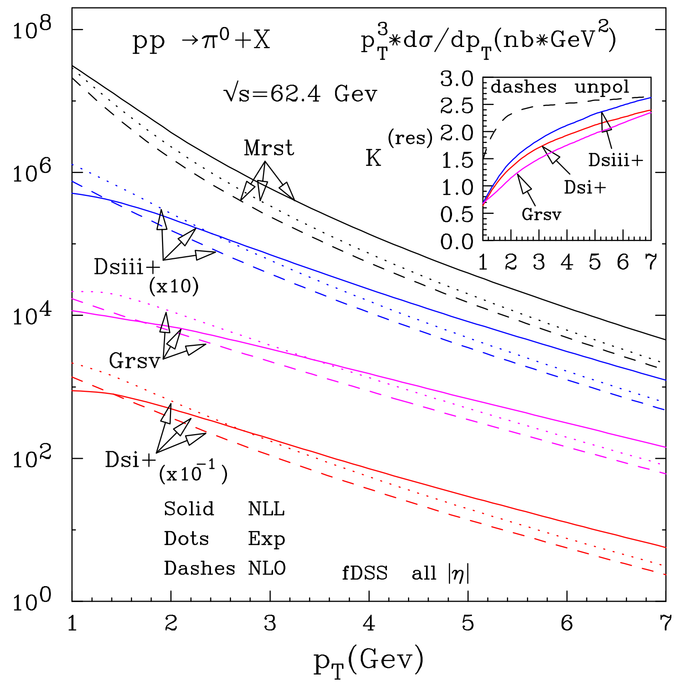

Figure 2: *Same as Fig. 1 but for* $\sqrt{S} = 62.4$ *GeV.*

of the resummed cross sections to the NLO cross sections (polarized or unpolarized) :

$$K^{(\mathrm{res})} = \frac{d\sigma^{(\mathrm{match})}/dp_T}{d\sigma^{(\mathrm{NLO})}/dp_T}\,. \tag{4.23}$$

As can be seen, $K^{(\mathrm{res})}$ is very large, meaning that resummation results in a dramatic enhancement over NLO. For the unpolarized case, this finding is in line with our previous results in [11]. It is interesting to see that $K^{(\mathrm{res})}$ is large also for all sets of spin-dependent parton distributions. It is evident, however, that the enhancement is somewhat smaller than in the unpolarized case. This immediately implies that the spin asymmetry A_{LL}^{π} will generally be reduced when going from NLO to the NLL resummed case. One also notices that the resummation effects vary slightly for the various polarized parton densities. This may be understood from the fact that Δg is of different size in the various sets. Typically, resummation effects are more important for partonic channels with more external gluons [11], so the size of Δg matters.

Fig. 2 shows similar results for $pp \to \pi^0 X$ at $\sqrt{S} = 62.4$ GeV. As expected from the fact that one is further away from threshold here, the soft-gluon approximation becomes somewhat less accurate in this case, in particular at the lower p_T. One can observe that the resummation effects are generally much smaller at $\sqrt{S} = 62.4$ GeV than at $\sqrt{S} = 19.4$ GeV.

As mentioned earlier, we have determined the resummed formulas for the fully rapidity-integrated cross section, whereas in experiments typically only a certain limited range in η is covered. In order to be able to compare to data, we therefore approximate the cross section (polarized or unpolarized) in the experimentally accessible rapidity region by

$$\frac{p_T^3\, d\sigma^{(\mathrm{match})}}{dp_T}(\eta\,\mathrm{in\,exp.\,range}) = K^{(\mathrm{res})}\,\frac{p_T^3\, d\sigma^{(\mathrm{NLO})}}{dp_T}(\eta\,\mathrm{in\,exp.\,range})\,, \tag{4.24}$$

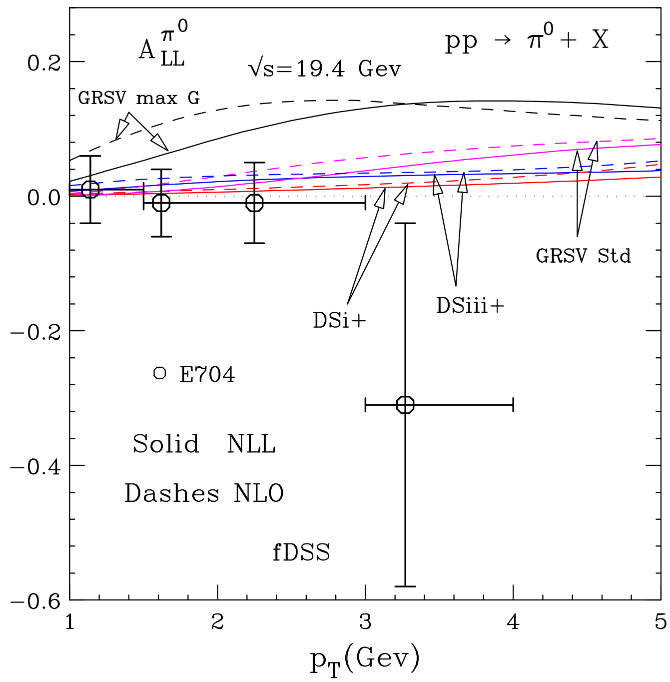

Figure 3: *Results for the double-spin asymmetry A_{LL}^{π} at NLO and for the NLL resummed case for various sets of polarized parton distributions, at $\sqrt{S} = 19.4$ GeV. We also show the experimental data of [8].*

where $K^{(\mathrm{res})}$ is as defined in Eq. (4.23) in terms of cross sections integrated over the full region of rapidity. In other words, we rescale the matched resummed result by the ratio of NLO cross sections integrated over the experimentally relevant rapidity region or over all η, respectively. [27]

Figure 3 shows our results for the spin asymmetry A_{LL}^{π} at $\sqrt{S} = 19.4$ GeV, for the NLO and NLL resummed cases, defined as in Eq. (2.3), averaged over the pion's Feynman-$x_F = x_T \sinh(\eta)$, $|x_F| \leq 0.1$. Again the scales have been chosen to be $\mu = p_T$. We also show the data by the Fermilab E704 experiment [8]. As expected from Fig. 1, A_{LL}^{π} generally decreases significantly from NLO to NLL. After NLL resummation, even a set with a very large Δg, such as the GRSV "maximal" scenario (which is now already ruled out by other measurements [5, 6, 7]) shows rough agreement with the data, given the rather large experimental uncertainties. It is interesting to note that similar results were found in [10] on the basis of LO studies invoking "intrinsic-k_T" effects.

We now return to the case of pp scattering at RHIC at $\sqrt{S} = 62.4$ GeV. Recently, first preliminary data for the spin-averaged high-p_T pion cross section as well as for the spin asymmetry A_{LL}^{π} were reported by the Phenix collaboration [16]. The data cover the pseudo-rapidity region $|\eta| \leq 0.35$. Fig. 4 compares our NLO and NLL resummed results for the spin-averaged cross section to the Phenix data. We use the scales $\mu = \zeta p_T$ with $\zeta = 1/2, 1, 2$. It is interesting to see that the data lie at the upper end of the rather wide NLO scale band, whereas the resummed predictions have a smaller scale dependence and tend to describe the data rather well with scale $\mu = p_T$. We remind the reader that at $\sqrt{S} = 200$ GeV the RHIC data are very well described by NLO with scale $\mu = p_T$ [5], while in the fixed-target regime resummation effects were found to

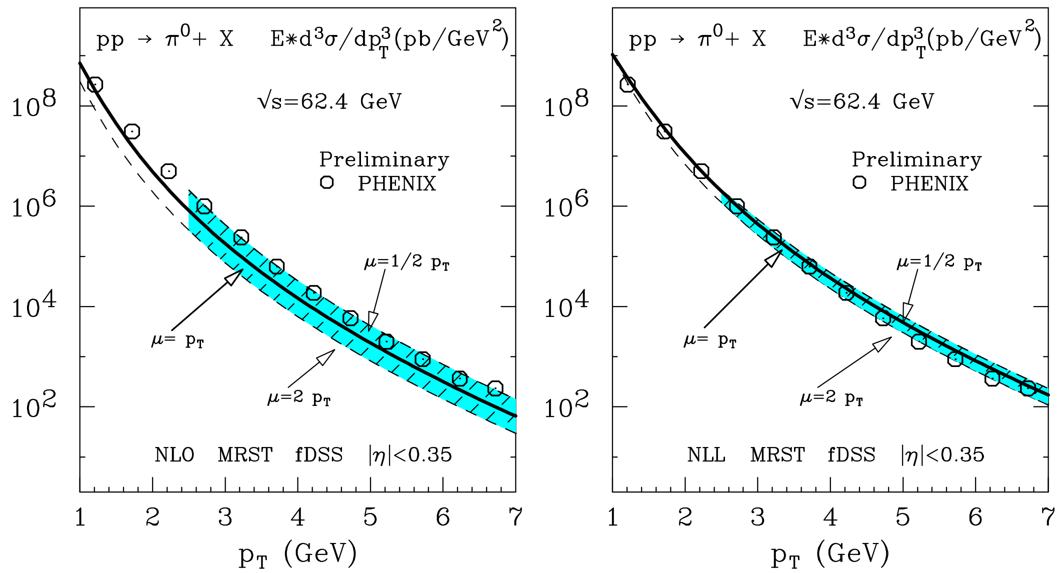

Figure 4: *Invariant cross section for* $pp \to \pi^0 X$ *at* $\sqrt{S} = 62.4$ *GeV at NLO and for the NLL resummed case. We also show the preliminary Phenix data [16].*

be very significant (see [11] and Fig. 1 above). We interpret all these features as indicating that threshold logarithms start to become relevant at $\sqrt{S} = 62.4$ GeV, which is "half way" between the typical fixed-target regime and RHIC's 200 GeV. Encouraged by the results in Fig. 4, we show in Fig. 5 our NLO and NLL results for the spin asymmetry A_{LL}^{π} at $\sqrt{S} = 62.4$ GeV, along with the Phenix data [17]. As can be seen in Fig. 5, sets with a large gluon polarization, like "GRSV max G", show a clear disagreement with the preliminary data. We observe that there is again a decrease of A_{LL}^{π} when going from NLO to NLL, but that the resummation effects are somewhat smaller than what we found at $\sqrt{S} = 19.4$ GeV. We have explicity checked that very similar results for the asymmetries are obtained when implementing other sets of fragmentation functions, like those from [28, 29].

5 Conclusions

We have studied in this paper the NLL resummation of threshold logarithms in the partonic cross sections relevant for the process $pp \to hX$ at high transverse momentum of the hadron h, when the initial protons are longitudinally polarized. We have found that, like for the spin-averaged case [11], the resummation effects are large for the spin-dependent cross section in the typical fixed-target regime at $\sqrt{S} \sim 20$ GeV. The spin asymmetry A_{LL}^{π} is significantly reduced by resummation. A phenomenological consequence is that the Fermilab E704 data [8] are compatible with essentially all currently sets of spin-dependent parton distribution functions, among them sets with a rather large gluon polarization.

We have also applied the resummation to the case $\sqrt{S} \sim 62.4$ GeV, at which recently preliminary data from RHIC have become available for both the spin-averaged cross section and the double-spin asymmetry [16, 17]. For this case sets with a very large gluon polarization in the range

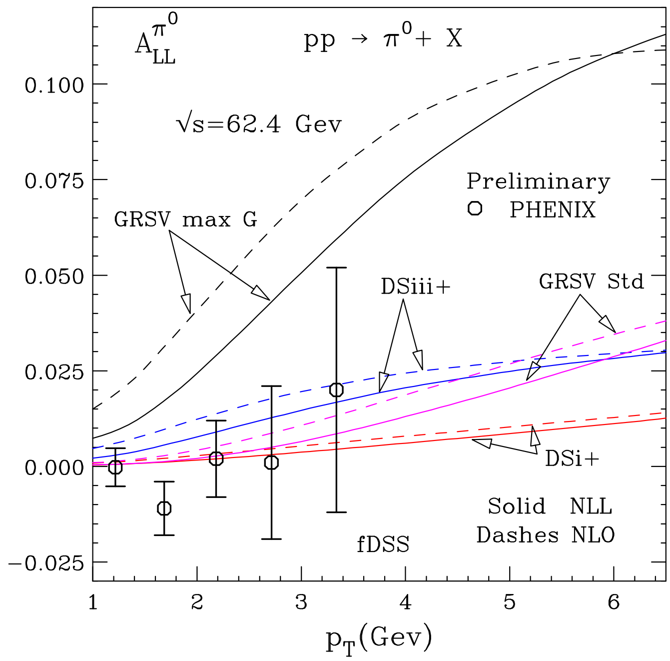

Figure 5: *Results for the double-spin asymmetry A_{LL}^{π} at NLO and for the NLL resummed case for various sets of polarized parton distributions, at $\sqrt{S} = 62.4$ GeV. We also show the preliminary experimental data of [17].*

$0.05 \lesssim x \lesssim 0.2$ are ruled out and distributions with a moderate to small gluon polarization are favored by the data. We find that resummation tends to lead to an improved description of the cross section. Its effect on the spin asymmetry is rather modest. We remind the reader that the threshold resummation is somewhat less reliable in this kinematic regime since one is further away from threshold than for the fixed-target case, so that subleading perturbative corrections may be more relevant. It will be desirable in the future to further improve the resummed calculation at $\sqrt{S} = 62.4$ GeV by including terms that are subleading near threshold. More reliable conclusions regarding the effects of threshold resummation on A_{LL}^{π} at this energy should then become possible.

Acknowledgments

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Appendix

A Results for the various subprocesses

In this appendix we compile the moment-space expressions for the spin-dependent Born cross sections for the various partonic subprocesses, and the polarized process-dependent coefficients $\Delta C^{(1)}_{ab\to cd}$, $\Delta G_{I\,ab\to cd}$ contributing to Eq. (3.10). As we mentioned in the main text, the coefficients $D_{I\,ab\to cd}$ are the same as in the unpolarized case; they have been given in Ref. [11]. Since the $\Delta C^{(1)}_{ab\to cd}$ have rather lengthy expressions, we only give their numerical values for $N_f = 5$ and the factorization and renormalization scales set to $\mu = Q$. In all expressions below, $C_A = 3$ and $C_F = (C_A^2 - 1)/2C_A = 4/3$.

$qq' \to qq'$:

$$\begin{aligned}
&\Delta\hat{\sigma}^{(\rm Born)}_{qq'\to qq'}(N) = \alpha_s^2 \frac{\pi C_F}{3C_A}\left(3N^2+5N\right) B\left(N,\frac{5}{2}\right) ,\\
&\Delta G_{1\,qq'\to qq'} = G_{1\,qq'\to qq'}, \quad \Delta G_{2\,qq'\to qq'} = G_{2\,qq'\to qq'} ,\\
&\Delta C^{(1)}_{1\,qq'\to qq'} = 17.9311 \; (N_f = 5) \, .
\end{aligned} \tag{A.1}$$

$q\bar{q}' \to q\bar{q}'$:

$$\begin{aligned}
&\Delta\hat{\sigma}^{(\rm Born)}_{q\bar{q}'\to q\bar{q}'}(N) = \alpha_s^2 \frac{\pi C_F}{3C_A}\left(3N^2+5N\right) B\left(N,\frac{5}{2}\right) ,\\
&\Delta G_{1\,q\bar{q}'\to q\bar{q}'} = G_{1\,q\bar{q}'\to q\bar{q}'}, \quad \Delta G_{2\,q\bar{q}'\to q\bar{q}'} = G_{2\,q\bar{q}'\to q\bar{q}'} ,\\
&\Delta C^{(1)}_{1\,q\bar{q}'\to q\bar{q}'} = 20.7021 \; (N_f = 5) \, .
\end{aligned} \tag{A.2}$$

$q\bar{q} \to q'\bar{q}'$:

$$\begin{aligned}
\Delta\hat{\sigma}^{(\rm Born)}_{q\bar{q}\to q'\bar{q}'}(N) &= -\hat{\sigma}^{(\rm Born)}_{q\bar{q}\to q'\bar{q}'}(N),\\
\Delta G^{(1)}_{1\,q\bar{q}\to q'\bar{q}'} = G^{(1)}_{1\,q\bar{q}\to q'\bar{q}'}, &\qquad \Delta C^{(1)}_{1\,q\bar{q}\to q'\bar{q}'} = C^{(1)}_{1\,q\bar{q}\to q'\bar{q}'} .
\end{aligned} \tag{A.3}$$

$qq \to qq$:

$$\begin{aligned}
&\Delta\hat{\sigma}^{(\rm Born)}_{qq\to qq}(N) = \alpha_s^2 \frac{2\pi C_F}{3C_A^2}\left(C_A(3N^2+5N) - 2N(3+2N)\right) B\left(N,\frac{5}{2}\right) ,\\
&\Delta G_{1\,qq\to qq} = 7/5\,, \quad \Delta G_{2\,qq\to qq} = -2/5\,, \quad \Delta C^{(1)}_{1\,qq\to qq} = 14.5364 \; (N_f = 5) \, .
\end{aligned} \tag{A.4}$$

$q\bar{q} \to q\bar{q}$:

$$\begin{aligned}
&\Delta\hat{\sigma}^{(\rm Born)}_{q\bar{q}\to q\bar{q}}(N) = \alpha_s^2 \frac{\pi C_F}{15C_A^2} N\left(C_A(2+N)(11+5N) - (N+3)(5+2N)\right) B\left(N,\frac{7}{2}\right) ,\\
&\Delta G_{1\,q\bar{q}\to q\bar{q}} = -3/13\,, \quad \Delta G_{2\,q\bar{q}\to q\bar{q}} = 16/13\,, \quad \Delta C^{(1)}_{1\,q\bar{q}\to q\bar{q}} = 24.2465 \; (N_f = 5) \, .
\end{aligned} \tag{A.5}$$

$q\bar{q} \to gg$:

$$\Delta\hat{\sigma}^{(\mathrm{Born})}_{q\bar{q}\to gg}(N) = -\hat{\sigma}^{(\mathrm{Born})}_{q\bar{q}\to gg}(N), \qquad \Delta C^{(1)}_{1\,q\bar{q}\to gg} = C^{(1)}_{1\,q\bar{q}\to gg},$$
$$\Delta G_{1\,q\bar{q}\to gg} = G_{1\,q\bar{q}\to gg}, \quad \Delta G_{2\,q\bar{q}\to gg} = G_{2\,q\bar{q}\to gg}\,. \tag{A.6}$$

$qg \to qg$:

$$\Delta\hat{\sigma}^{(\mathrm{Born})}_{qg\to qg}(N) = \alpha_s^2 \frac{\pi}{6C_A}\left(C_F + 2C_A\right)(3N^2+5N)B\left(N, \frac{5}{2}\right)\,,$$
$$\Delta G_{1\,qg\to qg} = G_{1\,qg\to qg}, \;\; \Delta G_{2\,qg\to qg} = G_{2\,qg\to qg}, \;\; \Delta G_{3\,qg\to qg} = G_{3\,qg\to qg}\,,$$
$$\Delta C^{(1)}_{1\,qg\to qg} = 14.2048\ (N_f = 5)\,. \tag{A.7}$$

$qg \to gq$:

$$\Delta\hat{\sigma}^{(\mathrm{Born})}_{qg\to gq}(N) = \alpha_s^2 \frac{\pi}{6C_A}\left(C_F + 2C_A\right)(3N^2+5N)B\left(N, \frac{5}{2}\right)\,,$$
$$\Delta G_{1\,qg\to gq} = G_{1\,qg\to gq}, \;\; \Delta G_{2\,qg\to gq} = G_{2\,qg\to gq}, \;\; \Delta G_{3\,qg\to gq} = G_{3\,qg\to gq}\,,$$
$$\Delta C^{(1)}_{1\,qg\to gq} = 21.2354\ (N_f = 5)\,. \tag{A.8}$$

$gg \to gg$:

$$\Delta\hat{\sigma}^{(\mathrm{Born})}_{gg\to gg}(N) = \alpha_s^2 \frac{\pi C_A}{15C_F}\left(21N^3 + 89N^2 + 92N\right)B\left(N, \frac{7}{2}\right)\,,$$
$$\Delta G_{1\,gg\to gg} = G_{1\,gg\to gg}, \;\; \Delta G_{2\,gg\to gg} = G_{2\,gg\to gg}, \;\; \Delta G_{3\,gg\to gg} = G_{3\,gg\to gg}\,,$$
$$\Delta C^{(1)}_{1\,gg\to gg} = 20.3233\ (N_f = 5)\,. \tag{A.9}$$

$gg \to q\bar{q}$:

$$\Delta\hat{\sigma}^{(\mathrm{Born})}_{gg\to q\bar{q}}(N) = -\hat{\sigma}^{(\mathrm{Born})}_{gg\to q\bar{q}}(N), \qquad \Delta C^{(1)}_{1\,gg\to q\bar{q}} = C^{(1)}_{1\,gg\to q\bar{q}},$$
$$\Delta G_{1\,gg\to q\bar{q}} = G_{1\,gg\to q\bar{q}}, \quad \Delta G_{2\,gg\to q\bar{q}} = G_{2\,gg\to q\bar{q}}\,. \tag{A.10}$$

In the above expressions, $B(a,b)$ is the Euler Beta-function.

B Spin-dependent color-connected Born cross sections

In this appendix we compile the color-connected Born cross sections that we need for our study. Our choices for the color bases follow precisely [13]. For a given color basis, the color-connected Born cross sections will appear as a matrix that we shall refer to as "hard matrix". We will not repeat the expressions for the soft matrices S, and the anomalous dimension matrices Γ in these bases, which may all be found in [13]. Like [13], we present our results for arbitrary partonic rapidity, even though for our actual study of the rapidity-integrated cross section we only need the case $\hat{\eta} = 0$. For each partonic reaction $ab \to cd$, we define the Mandelstam variables $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$, $u = (p_a - p_d)^2$. t, u are functions of $\hat{\eta}$. In all expressions below, $N_c = 3$ and $C_F = (N_c^2 - 1)/2N_c = 4/3$.

We begin with the quark-antiquark annihilation processes. Depending on the quark flavor, there are three different quark-antiquark subprocesses to consider, $q_j\bar{q}_j \to q_j\bar{q}_j$, $q_j\bar{q}_j \to q_k\bar{q}_k$ and $q_j\bar{q}_k \to q_j\bar{q}_k$. Each of these have their own hard matrix elements:

$q_j\bar{q}_j \to q_j\bar{q}_j$

$$\begin{aligned}
\Delta H_{11}^{q_j\bar{q}_j\to q_j\bar{q}_j} &= -H_{11}^{q_j\bar{q}_j\to q_j\bar{q}_j} = -\alpha_s^2\frac{2C_F^2}{N_c^4}\frac{(t^2+u^2)}{s^2}\,,\\
\Delta H_{12}^{q_j\bar{q}_j\to q_j\bar{q}_j} &= -H_{12}^{q_j\bar{q}_j\to q_j\bar{q}_j} = -\alpha_s^2\frac{2C_F}{N_c^3}\left[-\frac{(t^2+u^2)}{N_c s^2}+\frac{u^2}{st}\right],\\
\Delta H_{22}^{q_j\bar{q}_j\to q_j\bar{q}_j} &= \alpha_s^2\frac{1}{N_c^2}\left[-\frac{2}{N_c^2}\frac{(t^2+u^2)}{s^2}+2\frac{(s^2-u^2)}{t^2}+\frac{4}{N_c}\frac{u^2}{st}\right].
\end{aligned}\tag{B.1}$$

where $H_{ij}^{ab\to cd}$ refers in each case to the corresponding hard matrix for the unpolarized case given in [13].

$q_j\bar{q}_j \to q_k\bar{q}_k$

Here one has simply:

$$\Delta H^{q_j\bar{q}_j\to q_k\bar{q}_k} = -H^{q_j\bar{q}_j\to q_k\bar{q}_k}\,.\tag{B.2}$$

$q_j\bar{q}_k \to q_j\bar{q}_k$

The polarized hard matrix at lowest order can be expressed as,

$$\Delta H^{q_j\bar{q}_k\to q_j\bar{q}_k} = \alpha_s^2\begin{bmatrix} 0 & 0 \\ 0 & 2(s^2-u^2)/(N_c^2t^2)\end{bmatrix}.\tag{B.3}$$

There are only two different quark-quark processes to consider, depending on the quark flavors:

$q_jq_k \to q_jq_k$

Here the result is the same as for the process $q_j\bar{q}_k \to q_j\bar{q}_k$, but with the color basis inverted (equivalent to an interchange of the entries in the first and second rows and columns, $c_1 \to c_2$).

$q_jq_j \to q_jq_j$

Here,

$$\begin{aligned}
\Delta H_{11}^{q_jq_j\to q_jq_j} &= \alpha_s^2\frac{2}{N_c^2}\left[\frac{(s^2-u^2)}{t^2}+\frac{1}{N_c^2}\frac{(s^2-t^2)}{u^2}-\frac{2}{N_c}\frac{s^2}{tu}\right],\\
\Delta H_{12}^{q_jq_j\to q_jq_j} &= \alpha_s^2\frac{2C_F}{N_c^4}\left[N_c\frac{s^2}{tu}-\frac{(s^2-t^2)}{u^2}\right],\\
\Delta H_{22}^{q_jq_j\to q_jq_j} &= \alpha_s^2\frac{2C_F^2}{N_c^4}\frac{(s^2-t^2)}{u^2}\,.
\end{aligned}\tag{B.4}$$

$q_j q_k \to q_j q_k$

In the spin-dependent case one has:

$$\Delta H^{q_j q_k \to q_j q_k} = \alpha_s^2 \left[\begin{array}{cc} 2(s^2-u^2)/(N_c^2 t^2) & 0 \\ 0 & 0 \end{array} \right] . \tag{B.5}$$

$q\bar{q} \to gg$

Here,

$$\Delta H^{q\bar{q} \to gg} = -H^{q\bar{q} \to gg} . \tag{B.6}$$

$gg \to q\bar{q}$

Again, the polarized coefficients are the negatives of the unpolarized ones:

$$\Delta H^{gg \to q\bar{q}} = -H^{gg \to q\bar{q}} . \tag{B.7}$$

$qg \to qg$

Here,

$$\Delta H_{ij}^{qg \to qg} = \frac{s^2-u^2}{s^2+u^2} H_{ij}^{qg \to qg} . \tag{B.8}$$

Finally, we consider gluon-gluon scattering. For simplicity, we set $N_c = 3$ explicitly here.

$gg \to gg$

$$\Delta H_{ij}^{gg \to gg} = \frac{s^4-t^4-u^4}{s^4+t^4+u^4} H_{ij}^{gg \to gg} . \tag{B.9}$$

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